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EFFECTS OF UNIFORM TARGET DENSITY
ON RANDOM SEARCH

by

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Three different types of boundary reflection patterns were analyzed for their ability to create a uniform target density. Two of those patterns were successful, but only one was suitable for further analysis. Support for the hypothesis was achieved when the PND (t) using this new uniform density target motion model was found to be of exponential form. It was also discovered that the exponential detection rates for this new simulation model were very close to the detection rates predicted by B. O. Koopman's random search formula

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Effects of Uniform Target Density on Random Search

by

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ABSTRACT

This thesis was motivated by a study performed by Commander Submarine Force Pacific (COMSUBPAC) of detection rates for a random search model. COMSUBPAC's study concluded that the probability of nondetection to time t, PND(t), was not of the form exp $(-\gamma\ t)$, as believed by many. The target motion model used for that study was a new and interesting model, therefore this investigation began by analyzing that model. This investigation discovered that the density of targets for COMSUBPAC's motion model was not uniform over the search area, which might lead to a nonexponential form for PND(t). To lend support to the hypothesis that a uniform distribution of target position can lead to an exponential form of PND(t) the target motion was altered to achieve a uniform target density. The same basic target motion was used because of its inherent advantages over other target motion models. Three different types of boundary reflection patterns were analyzed for their ability to create a uniform target density. Two of those patterns were successful, but only one was suitable for further analysis. Support for the hypothesis was achieved when the PND(t) using this new uniform density target motion model was found to be of exponential form. It was also discovered that the exponential detection rates for this new simulation model were very close to the detection rates predicted by B. O. Koopman's random search formula.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. DESCRIPTION OF HENZE'S NODE MODEL

A. INTRODUCTION

It is a very difficult, if not impossible, computational problem to calculate the probability of detection or mean time to detection for a stationary searcher against a target moving 'randomly', although some limited successes have been made. B. O. Koopman made several assumptions about the random target motion [Ref. 1] and achieved his well known result that the time to detection is an exponentially distributed random variable. J.N. Eagle has analyzed a Brownian motion target and found a closed form solution for the probability of nondetection to time t, PND(t), which involved an infinite sum of Bessel functions [Ref. 2: page 44]. He also discovered a more simple solution in exponential form as t becomes arbitrarily large. Previous thesis students at the Naval Postgraduate School (NPS) have examined popular motion models, such as the diffusing target and random tour target, using computer simulations [Refs. 3,4]. They concluded, for these target motion models, that the PND(t) had an approximate exponential form.

A new target motion model was recently introduced by J. Henze [Ref. 5]. Henze reported that Monte Carlo simulation using this model provided a time to detection which was distributed as a Pareto[1] random variable, rather than the exponential random variable many would have expected. We begin investigating this interesting and surprising result with a description of the Henze target motion model.

B. DESCRIPTION OF HENZE'S NODE MODEL

1. Search Area A

The target is constrained to a circular search area A. In many studies a square or rectangular region is specified, perhaps because boundary reflections (which are easier to model with linear area boundaries) are required. Henze also uses a square search area in his studies, but identical results can also be achieved with a circular search area. A circular area is radially symmetric and eliminates any target motion problems which might occur in the corners. For theoretical studies such as this, a

[1]The Pareto distribution considered in this thesis and Henze's study has a probability distribution function, $f(x)$, of the form: $f(x) = \gamma(1 + \gamma x/\beta)^{-(\beta+1)}$, where γ is the detection rate, R is the detection radius, A is the search area, v is the target speed, and $\gamma = 2Rv/A$. $\beta = 2$ in both studies, where β is the shape parameter.

circular area is more appropriate and will be the only geometry considered. In order to further simplify the geometry, A will be equal to π, which means that the search area radius, R_A, is equal to one.

2. **Motion of the Target**

The target moves randomly from point to point (or node to node) in a straight line at constant speed over the search area A (see Figure 1.1). The nodes are chosen randomly and independently from a bivariate uniform distribution of the search region

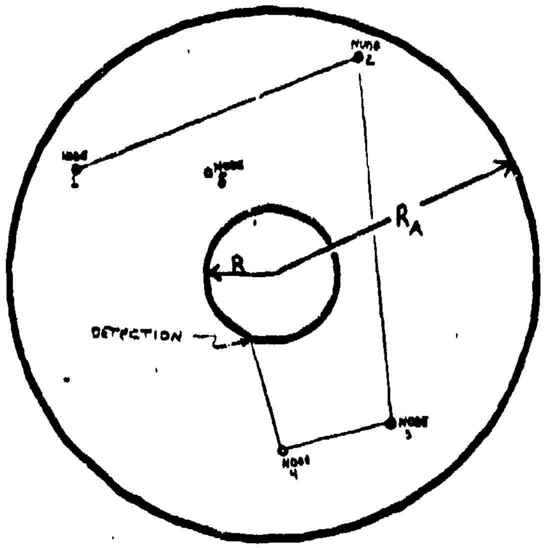

Figure 1.1 Henze's Simulation Geometry.

A. The motion is very simple and requires no reflections off area boundaries. There is also no need to estimate coefficients such as a diffusion constant or a rate of course change for the target, as required in other popular motion models [Refs. 3,4]. Thus the Henze target motion model appears very attractive for operational analysis of the search problem.

3. **Target Starting Position**

The target's starting position is uniformly distributed over the search region A and is selected independently from the same distribution as the target nodes.

4. **Target Velocity v**

Unless otherwise noted the target velocity v used in each simulation is always equal to one search area radius per unit of time t. This is a unit velocity with $v = 1$, which allows the results to be stated equivalently in terms of distance traveled *or* time traveled.

5. Searcher Location

The searcher is stationary for the entire search period, but for each repetition of the simulation the searcher's location is selected independently from the same bivariate uniform distribution as the target nodes.

6. Detection

The searcher has a deterministic detection capability over a disk of radius R (see Figure 1.1). The probability of detection inside this disk is equal to one, and therefore the searcher is said to have a 'cookie cutter' sensor with detection range R [Ref. 6: page 2-1]. Detection occurs the first time that the target enters the searcher's detection disk; that is, when the distance between the target and the searcher is less than R. In order to minimize the consideration of edge effects, the detection range R should be considerably smaller than R_A. Since $R_A = 1$, R is the ratio of detection radius to search area radius.

C. DESCRIPTION OF PROGRAM NODE

1. General

Program NODE, found in Appendix A, is a Monte Carlo simulation of the search scenario described above. Arbitrarily, the stationary platform is called the 'searcher', and the moving platform the 'target', where the target can be thought of as a submarine and the searcher as a sonobouy. NODE is coded in FORTRAN and is designed for use at the NPS. NODE uses the external subroutine LRND in the Non-International Mathematics and Statistics Library (NONIMSL) to generate uniform random variables and it uses the GRAFSTAT graphics system. NODE is an event driven simulation, which is much more efficient than a time step simulation. Instead of stepping through incremental time steps and then updating the situation, NODE evaluates the situation at the end of each target leg and analytically solves the equations for detection during the leg. As opposed to time stepping, this path is *not* approximated by a series of points. Therefore detection may occur exactly at the detection disk boundary, and it is not possible for the simulated target path to jump across the edge of the detection disk without achieving detection. This more elegant process also eliminates the potential inaccuracies involved with determining a time step increment. The program will also run much faster than a time step simulation. This eliminates the need to terminate the repetition after some TMAX which would also introduce inaccuracies.

2. **Inputs**

- Radius of detection disk R
- **Target speed** v
- Number of replications (NREP)

3. **Functioning of the Program**

a. At the beginning of each replication, the initial starting position of the target and the searcher is drawn from a bivariate uniform distribution over the area A.

b. The target's next position is drawn independently from the same distribution. These two points form a line for the search leg.

c. In order to determine if a detection has occurred on the search leg the equations for the search leg line

$$(Y-y_{new})^*(x_{old}-x_{new}) = (X-x_{new})^*(y_{old}-y_{new})$$

and for the searcher's detection disk

$$(X-x_{searcher})^2 + (Y-y_{searcher})^2 = R^2$$

are solved simultaneously for the coordinate (X,Y) of intersection where (x_{old}, y_{old}), (x_{new}, y_{new}) are coordinates for the beginning and ending search leg respectively and $(x_{searcher}, y_{searcher})$ are coordinates for the searcher.

d. If only an imaginary solution to the system of equations exists then there has been no detection and a new target position is drawn.

e. If there is a solution then it must be determined if the solution exists between the old and new target positions. If it does not then again there has been no detection and a new target position is drawn. If it does then a detection has occurred.

f. After each target node the time to detection counter is updated.

g. After each detection the replication terminates and the process continues until the specified number of replications is reached.

4. **Output**

For each replication the simulation output is the total time t which it took for the searcher to make a detection. This data is then read into GRAFSTAT from which graphs of PND(t) may be drawn and analyzed.

II. ANALYSIS OF HENZE'S NODE MODEL

A. INTRODUCTION

Koopman [Ref. 1: p. 6] argued that PND(t) is $\exp(-2Rvt/A)$, where v is the speed of the random search, R is the detection range, and A is the size of the area in which the random search is performed. Henze's results suggest that PND(t) is not exponential as suggested by Koopman. Instead it is more closely represented by a Pareto distribution. This apparent inconsistency is curious and requires some further investigation. This chapter will delve into Henze's search model and try to discover the reasons for the disagreement between Henze's results and Koopman's theories.

B. INVESTIGATING THE DEPENDENCE ON SEARCHER'S POSITION

1. Boundary Effect

As mentioned earlier, at the start of each repetition Henze chose the stationary searcher's position ($X_{searcher}$, $Y_{searcher}$) from a bivariate uniform distribution where

$$(X_{searcher})^2 + (Y_{searcher})^2 < A/\pi.$$

Selecting the searcher's position from this distribution allows part of the searcher's detection disk to be outside the search area boundary if

$$(X_{searcher})^2 + (Y_{searcher})^2 > (\sqrt{(A/\pi)} - R)^2.$$

When this situation occurs, the time to detection will be greater than otherwise, since the size of the detection disk is effectively reduced. By changing Henze's model to select the searcher's position from the bivariate uniform distribution where

$$(X_{searcher})^2 + (Y_{searcher})^2 < (\sqrt{(A/\pi)} - R)^2$$

a closer fit to the exponential distribution of PND(t) is achieved, although the Pareto distribution still provides the best fit. All further analysis of Henze's model includes this revision.

2. Radial Effect

a. Pareto and Exponential Chi-Square Values

In order to investigate how PND(t) varies with respect to searcher location, simulations were performed where the stationary searcher position was not selected from a uniform distribution, but was fixed at one point. Then for different simulation runs, this point was moved along the radius of the search area. By comparing the Chi-Square goodness-of-fit values assuming that the times to detection are distributed exponentially with the Chi-Square values assuming a Pareto distribution, we are able to determine how the fit of the PND(t) varies with searcher location. Remembering that the smaller Chi-Square value the better the fit, Figure 2.1 shows that the Pareto distribution is a better fit near the area boundaries, while the exponential distribution gives the best fit towards the center. From Figure 2.1, with $R=0.01$, it appears that the crossover point where the best fit distribution changes occurs approximately at 0.75. Then the fraction of the search area where the exponential distribution provides the best fit is $(0.75)^2=0.56$, approximately equal areas.

Note also from Figure 2.1 that when the searcher position is <0.75 the Pareto distribution provides a fit almost as good as that of the exponential distribution. However, when the searcher is >0.75 the Pareto fit is *much* better than the exponential fit. When the searcher's position is uniformly distributed about the search area, the extremely poor exponential fit near the area boundaries (totaling almost half of the entire area) helps explain why the Pareto distribution provided the best overall fit in Henze's model.

b. Steady State Target Density with no Searcher

Some insight can be gained by examining the steady state distribution of targets following the Henze motion model. Figure 2.2 is a scatter and radial empirical density plot of those positions when the searcher has been removed. Note that the scales for the X and Y axes are not identical in this scatter plot and others in this study. By observation it appears that the distribution of targets is not uniform. Instead, a target is more likely to be found in the center of the search area as opposed to the edge. By measuring the radial distance of each of the points from the center and weighting each point by the inverse of its radial distance we are able to find the density of targets as a function of distance from the center. Figure 2.2 distinctly shows that this density of targets is *not* uniform across the search area, which is a requirement of Koopman's random search formula. This observation could help explain Henze's results.

C. EXAMINING PREDICTED DETECTION RATES

Another interesting feature of Henze's model is that his detection rate does not approximate the detection rate which Koopman's random search formula would

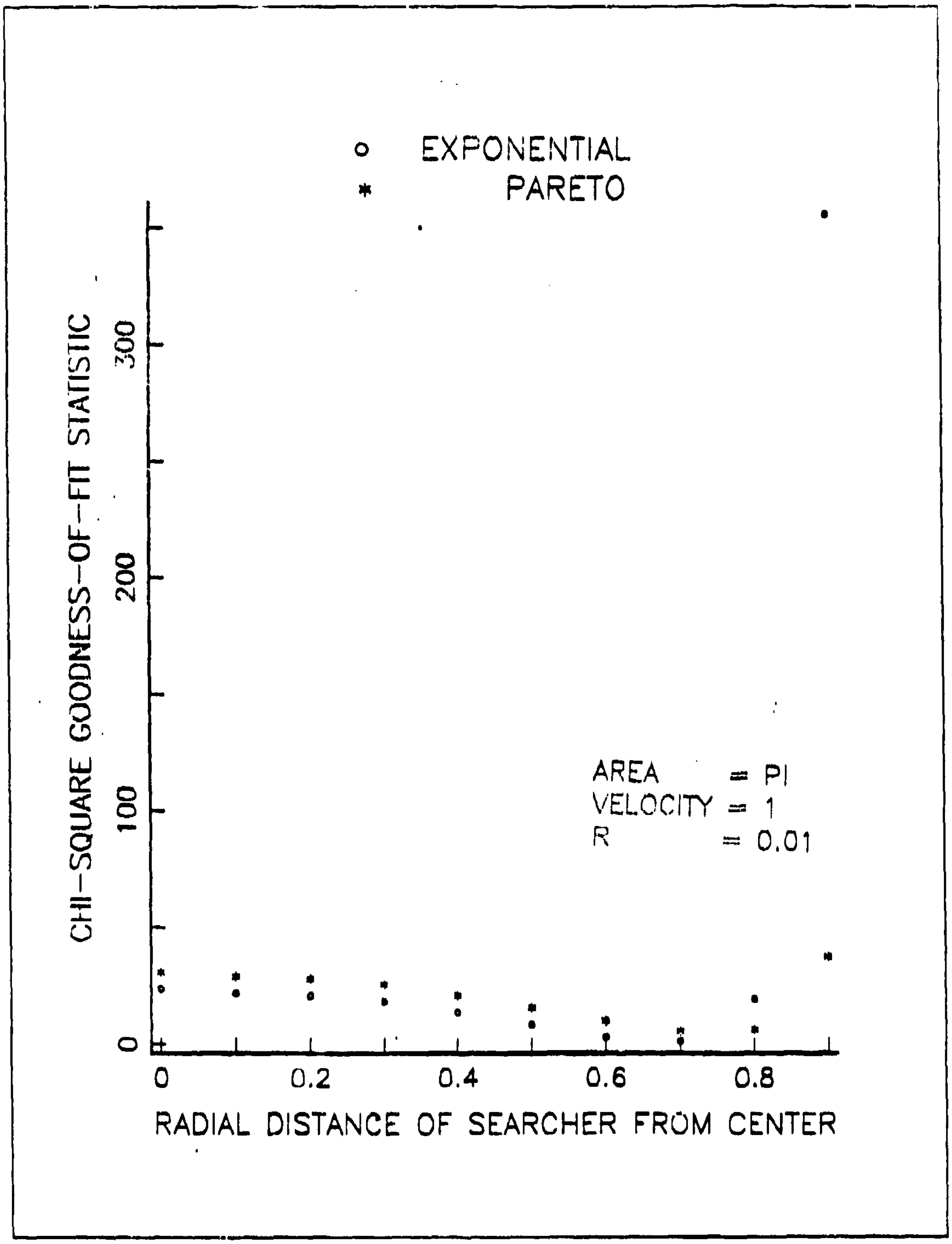

Figure 2.1 Comparison of Pareto vs. Exponential Fit for Different Searcher Locations.

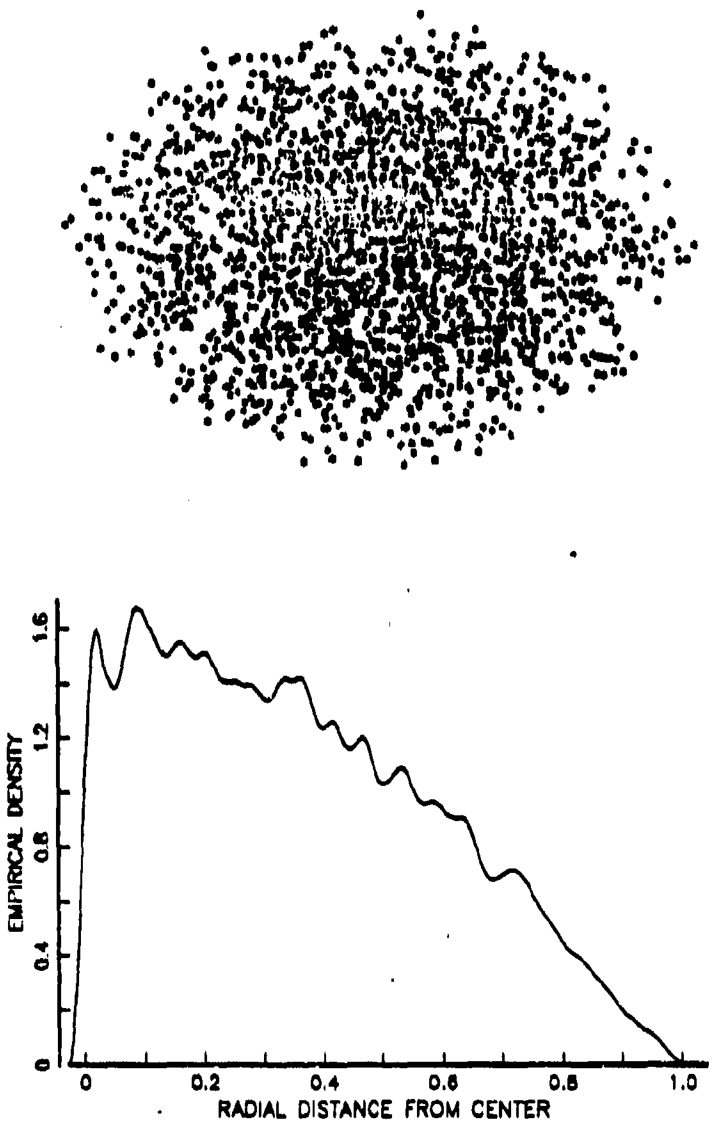

Figure 2.2 A Snapshot of Non Detected Targets at Steady State and their Empirical Density.

predict, even when the searcher is at the center of the area where the detection rate is close to exponential. As mentioned earlier, Koopman's random search model predicts a detection rate of $2R\,\nu/A$. The Henze target motion simulation included 10,000 iterations 10,000 at each searcher detection radius R. A plot of the PND(t) vs time (or distance) traveled for values of R from 0.001 to 0.03 is illustrated in Figure 2.3 Notice that the scale of the Y axis is logarithmic so exponential distributions will plot as straight lines and the search detection rate is the negative slope of these curves. The curves of Figure 2.3 are almost linear which implies that this distribution is approximately exponential. In his study, Henze notes that the Pareto approaches the exponential distribution as the Pareto parameter $\beta \rightarrow \infty$. Table 1 provides a comparison of Henze's target model detection rate determined by least squares fitting of the simulation data with Koopman's random search detection rate. Note that the

TABLE 1

COMPARISON OF DETECTION RATES FOR RANDOM SEARCH AND HENZE'S MODEL

Detection Radius R	Random Search Detection Rates (RS)	Simulation Detection Rates (S)	$\frac{S}{RS}$
0.001	6.366E-4	13.25E-4	2.08
0.003	1.910E-3	4.279E-3	2.24
0.006	3.820E-3	8.223E-3	2.15
0.01	0.006366	0.013955	2.19
0.03	0.01910	0.04186	2.19

detection rate for Henze's model is more than twice that predicted by Koopman. This might be explained by remembering that the target density for Henze's model was concentrated in the center of the search area, precisely where the searcher is located. This should result in more detections per unit time; i.e., a higher detection rate.

It appears as if a nonuniform steady state target distribution may be a primary reason why Henze's model does not produce random search results. Our next task will be to modify Henze's target motion to achieve a uniform target distribution and determine if the random search formula holds in that situation.

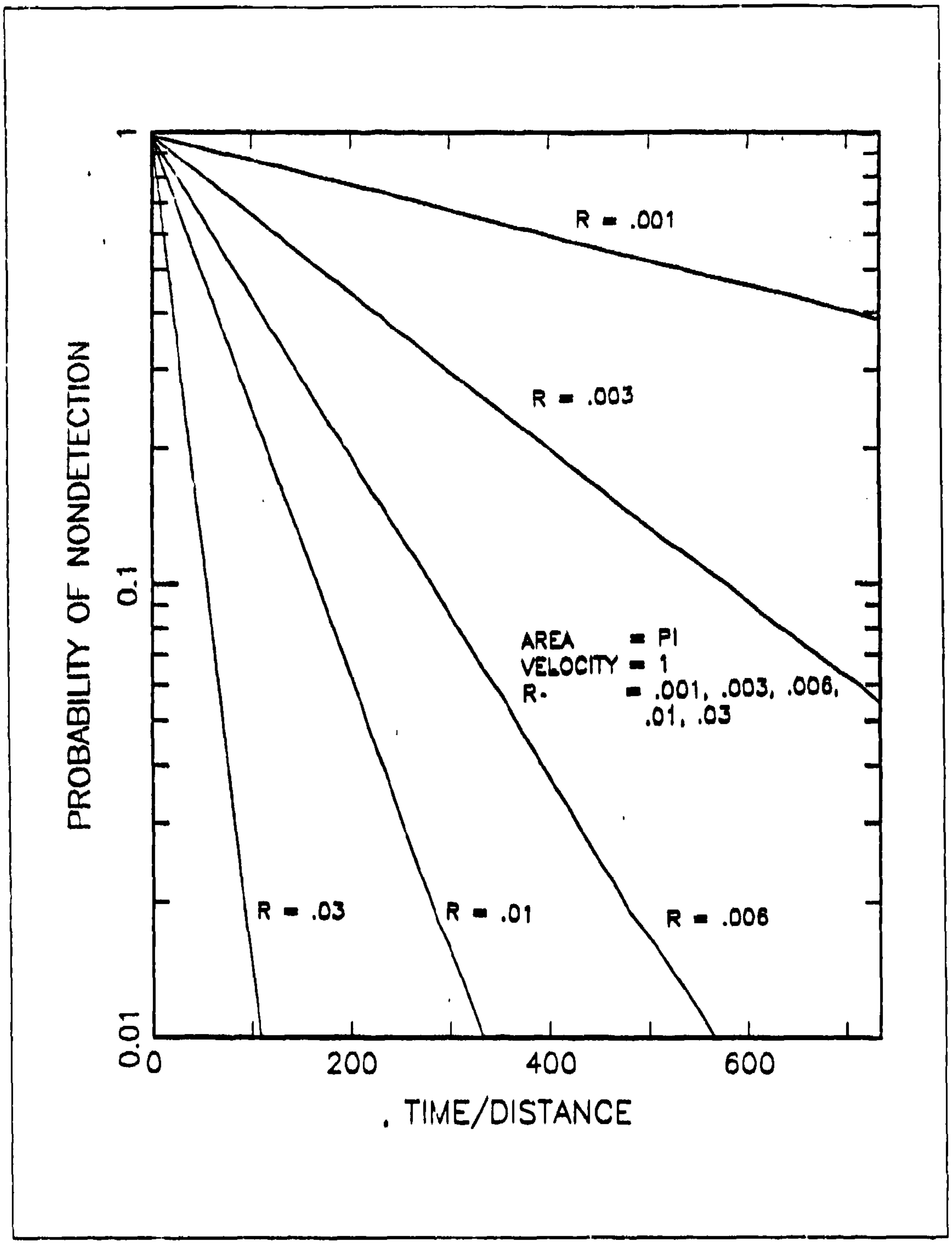

Figure 2.3 Probability of Non Detection by Time t for different Values of Detection Radius R.

III. RESTORING THE UNIFORM DISTRIBUTION OF TARGETS

A. INTRODUCTION

Prior thesis students [Refs. 3,4] have shown that with diffusion and random tour target motion models, Koopman's random search formula is a reasonable approximation. It would be useful to make the same connection using Henze's motion model, since his model has the important advantages which have been previously discussed. The benefits of his model, such as simplicity and the use of operationally meaningful variables, should be retained in any revised model. But this revised model must have one feature which Henze's model is lacking. It must have a uniform steady state distribution of targets in order to meet Koopman's assumptions of random search. This assumption of Koopman's is more explicitly stated by Washburn [Ref. 6: page 2-6].

We have seen that the target density in Henze's model was concentrated in the center of the search area. This central tendency of targets is caused by the manner in which target turnpoints (nodes) are selected. By using Henze's method of target motion, the probability that a target path will ever reach the search area boundary is 0. Whereas in a model which permits boundary reflections (such as the diffusing and random tour models examined in [Refs. 3,4]) there are many target paths which intersect the search area boundary. Therefore, in order to reduce the central tendency of targets in Henze's model, it may help to include boundary reflections. This chapter will investigate different types of target reflection to restore a uniform target distribution to Henze's model. There is no target involvement with the searcher in this chapter.

B. PERFECT REFLECTION MODEL

1. Description of a Perfect Reflection Path

A perfect boundary reflection (also known as specular reflection) is one in which the angle of incidence is equal to the angle of reflection with respect to the boundary normal. This is illustrated in Figure 3.1. If the only influence on target motion in a circular area is perfect boundary reflection, then these angles are equal for every reflection. Figure 3.2 shows several possible target paths for different reflection angles. It is interesting to note that if the target does not pass through the center of A on its first leg, it never will. Additionally, the minimum distance from any leg to the center is the same for all legs.

2. **Determining Target Density**

Whenever we determine a target density in this study it is important to ensure that the target is in steady state. By allowing the target to travel some 'long' distance, say 50 area radii ($50R_A$), we assume that the target distribution is in steady state, if a steady state distribution exists. After the target is in steady state, we record its

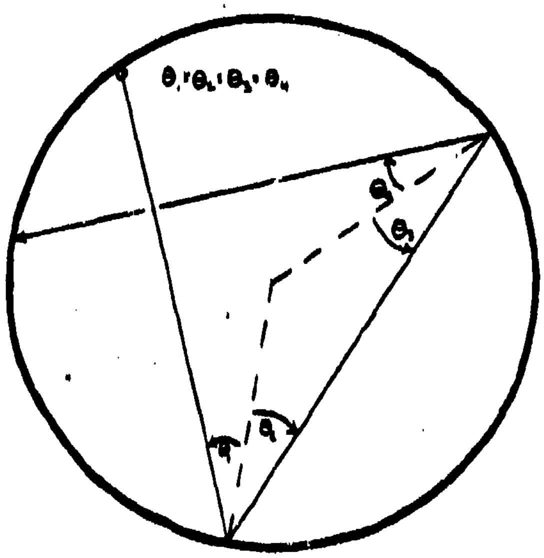

Figure 3.1 Perfect Reflection Geometry.

position and then repeat this exercise for many targets. Eventually the empirical density is developed.

3. **Target Starting Position Is Uniformly Distributed on the Area Circumference**

a. Creating the Target Motion

To create this perfect reflection motion, uniformly choose any two points between 0 and 2π on the circumference of the search area. Then let these two points define the first target leg and allow perfect reflection to determine the subsequent target motion. This motion should have more of the target density at the area boundaries than Henze's model due to these boundary reflections.

b. Analyzing Target Density

(1) *Target Stop Time is Deterministic.* Figures 3.3a and 3.3b display the scatter plots and empirical density plots of target positions for five different stopping

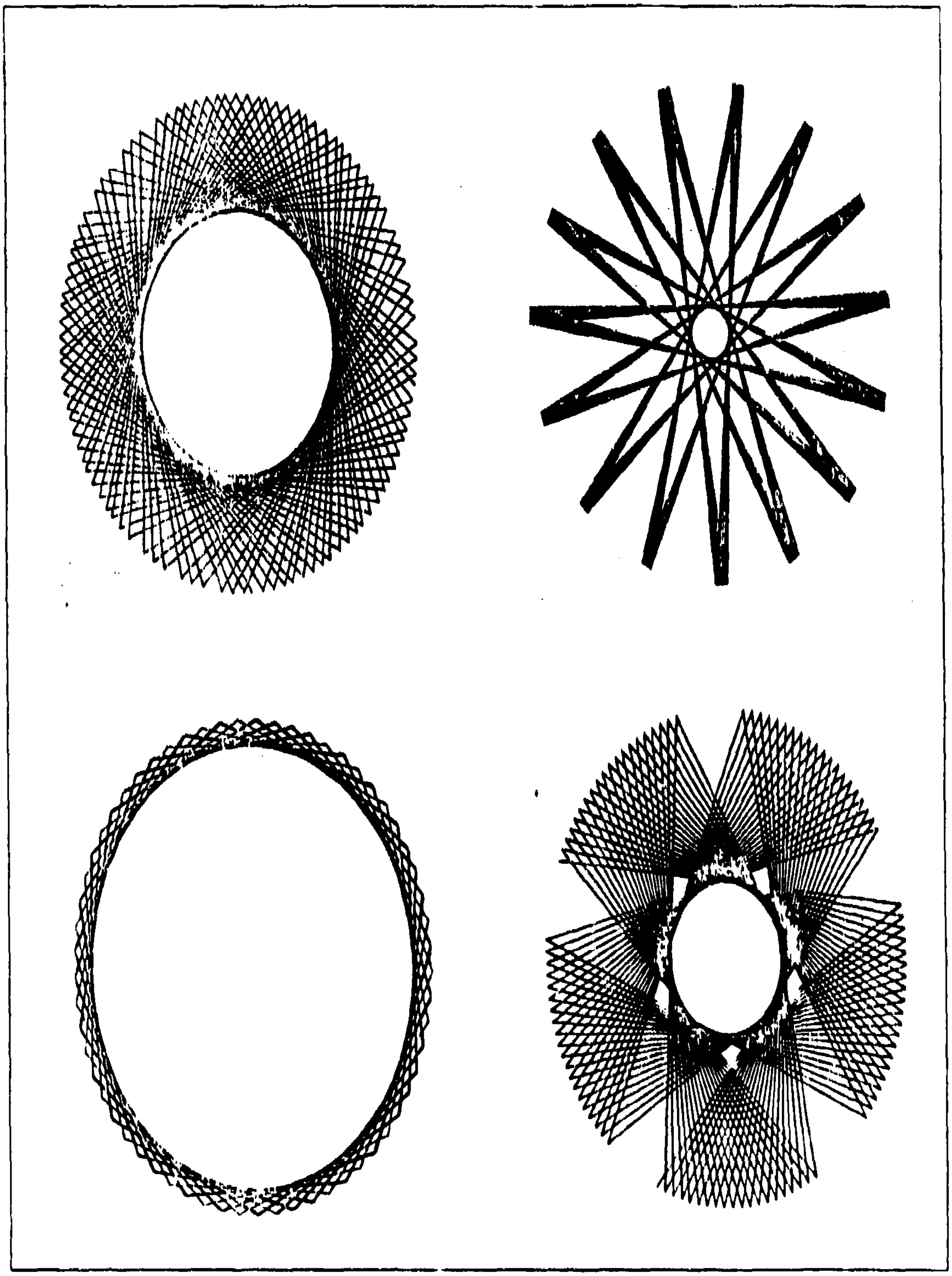

Figure 3.2 Possible Paths for a Perfect Reflecting Target.

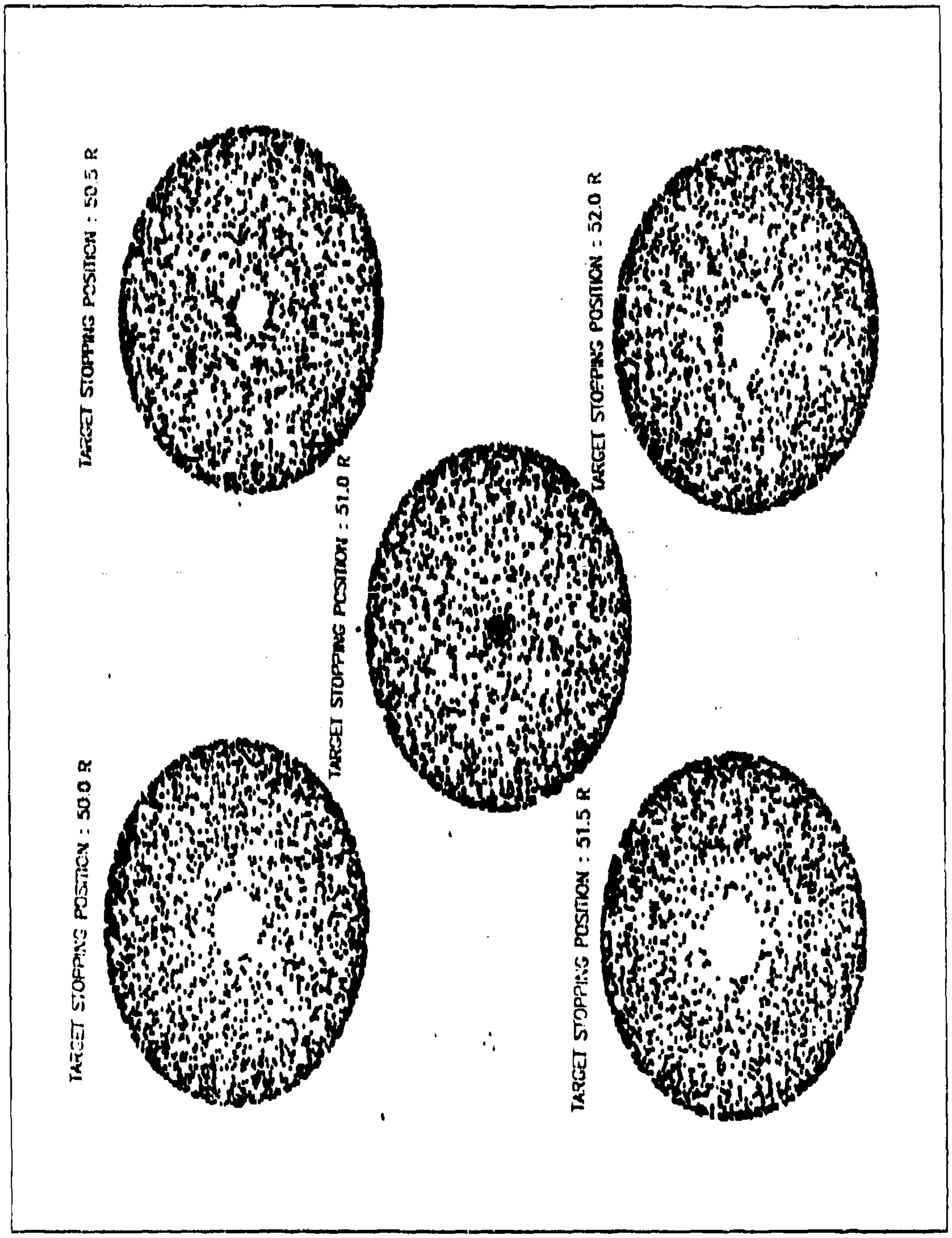

Figure 3.3a Scatter Plots of Perfect Reflecting Targets Starting on the Circumference for Stopping Times $50R_A$ to $52R_A$.

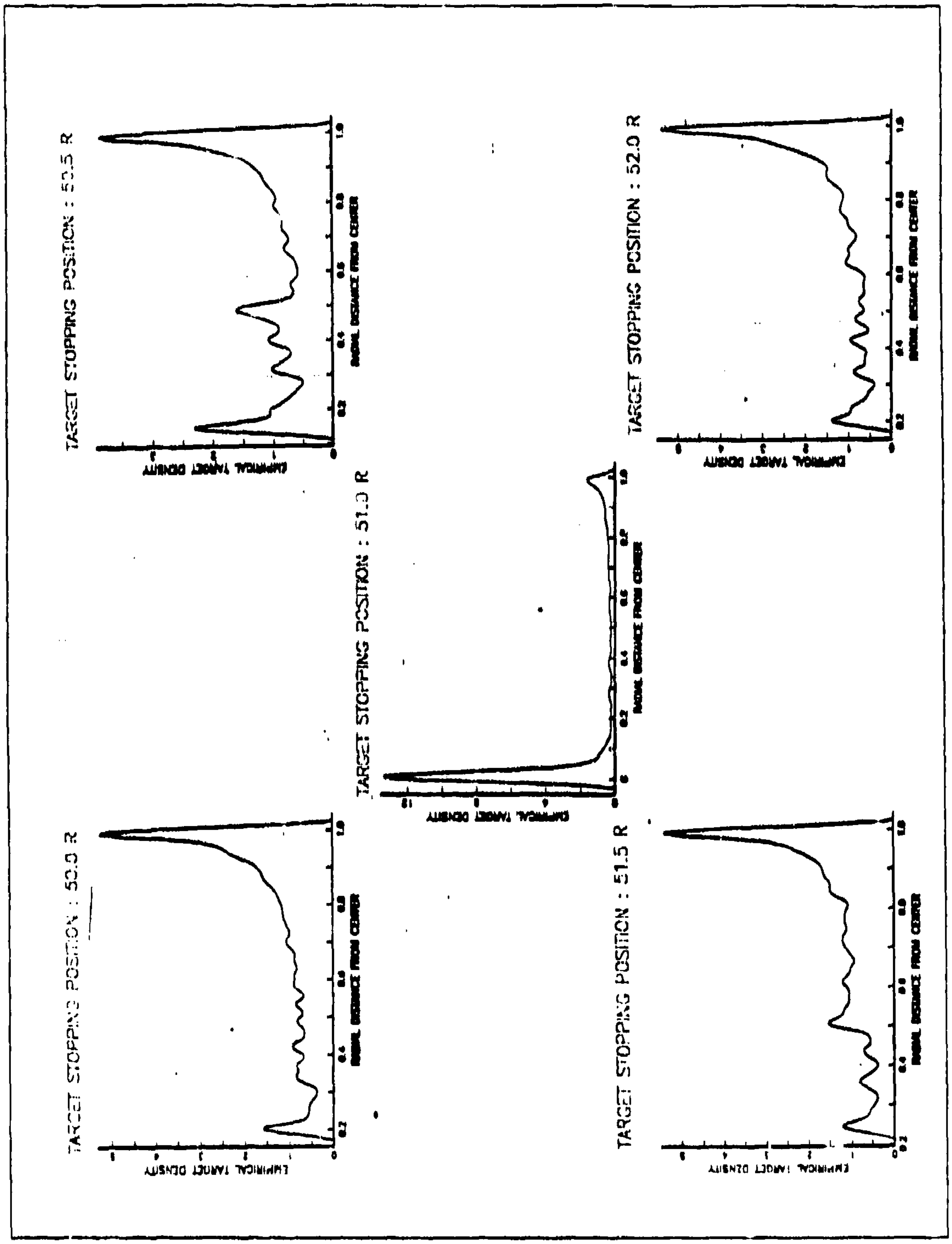

Figure 3.3b Empirical Density Plots of Perfect Reflecting Targets Starting on the Circumferen for Stopping Times 50R to 52R.

times These stop times correspond to a path length of $50R_A$ to $52R_A$ and should be long enough to achieve a steady state distribution, if such a distribution exists. Note from Figures 3.3a and 3.3b that the target density is dependent on the target stopping time. As the stopping time varies from $50R_A$ to $52R_A$ the distribution completes one full cycle. Note that on every even multiple of R_A the density is concentrated on the boundary and on every odd multiple of R_A the density is concentrated at the center. This process is apparently cyclic and does not have a steady state distribution. This feature makes this target motion unacceptable for our purposes.

(2) *Target Stop Time is Random.* The same perfect reflection simulation is executed but this time for each repetition the target stop time is uniformly selected between $50R_A$ and $52R_A$. When the stopping time is uniformly selected between one cycle of the target steady state distribution, a mean value of the stopping time cycle is produced which should generate a pseudo-stationary condition. Figure 3.4 illustrates that the target density for this pseudo-stationary condition is still not uniform, which requires us to investigate another motion model with a different set of conditions.

4. Target Starting Position is Uniformly Distributed in the Search Area

a. Creating the Target Motion

Now the target starting position is uniformly distributed over the entire search area, and its initial direction of motion is uniformly distributed between 0 and 2π. When the target intersects the boundary, its next leg is determined by perfect reflection.

b. Analyzing Target Density

Figure 3.5 illustrates that when the perfectly reflecting target starts uniformly in the entire search area and chooses a uniform direction to begin searching, the uniform target density is apparently reclaimed. Now that we have found the distribution which we were seeking it would be interesting to see if this target distribution may be achieved by any other target motion.

C. UNIFORM REFLECTION MODEL

1. Description of Uniform Reflection

A uniform boundary reflection is one in which the target's scatter angle θ is not dependent on the angle of incidence but is uniformly distributed between $-\pi/2$ and $\pi/2$ from the boundary normal as suggested in Figure 3.6. Figure 3.7 is a sample path for this type of target motion.

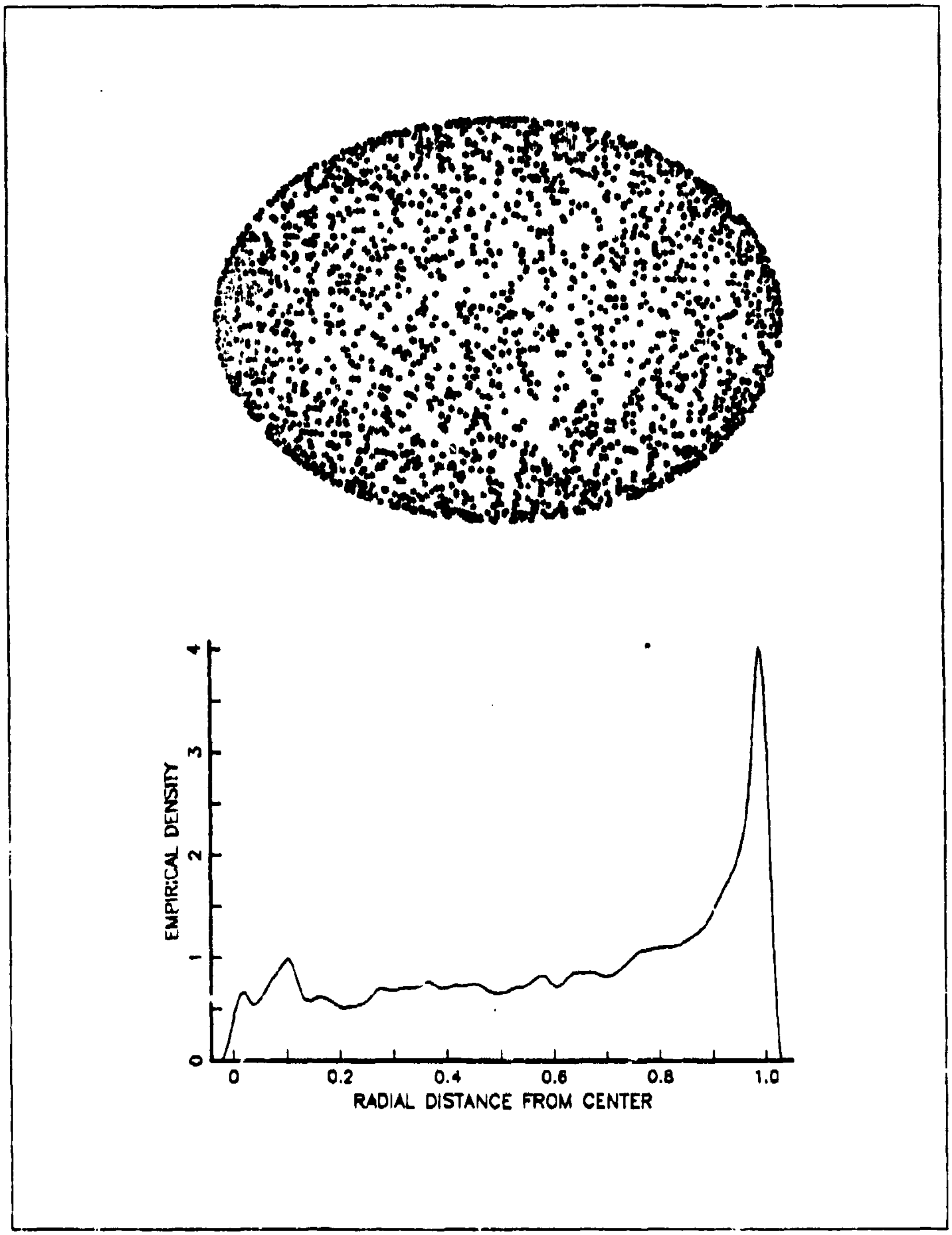

Figure 3.4 Perfect Reflecting Target Starting on the Circumference
Stopping Time is Randomly Selected between 50R and 52R.

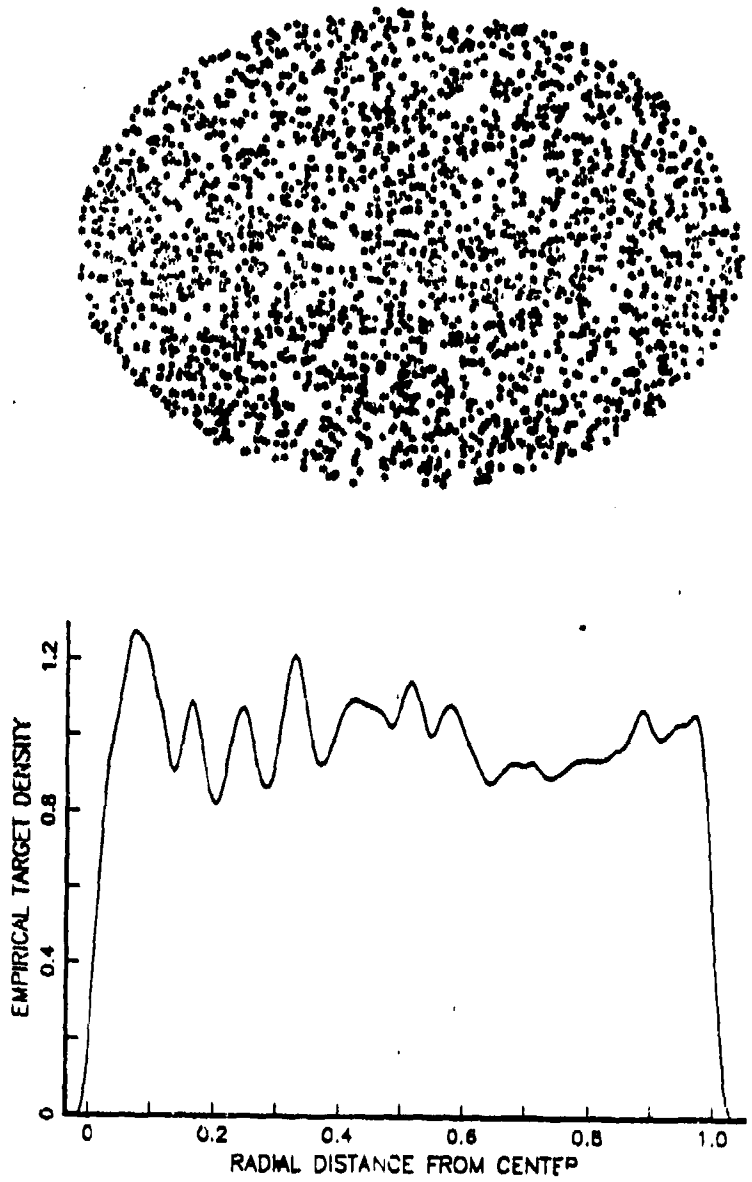

Figure 3.5 Perfect Reflecting Target when Starting Uniformly in the Search Area.

2. **Connection between Uniform Reflection Model and Henze's Node Model**

Another way to describe this type of target motion is very similar to Henze's original node model. If each target turnpoint is randomly selected uniformly between 0 and 2π on the circumference of the search area, instead of choosing a uniform

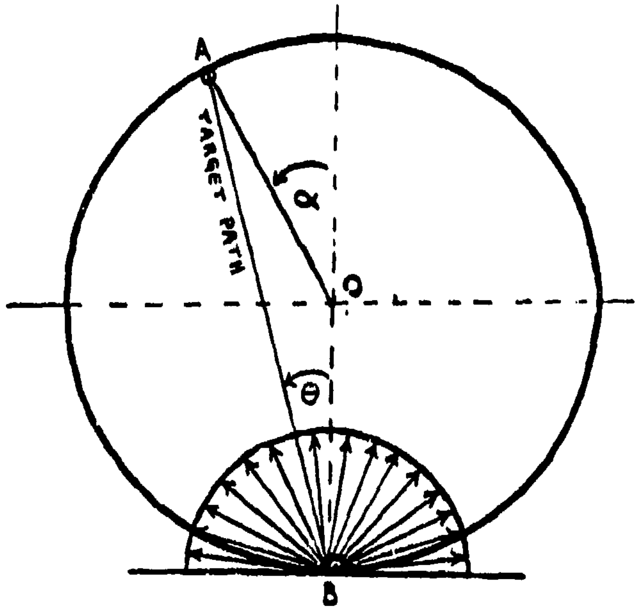

Figure 3.6 Uniform Reflection Geometry.

reflection angle θ, then we have a situation identical to uniform reflection motion. It is a simple transformation of variables to show that selecting a uniform reflection angle θ is equivalent to selecting a uniform point α on the search area circumference. To prove this consider Figure 3.6. Since Δ OAB is isosceles Angle OBA = Angle OAB = θ and $\alpha = 2\theta$. If θ is a random variable and is distributed between $-\pi/2$ and $\pi/2$ or notationally, $\theta \sim U[-\pi/2, \pi/2]$ then

$$\alpha = 2\theta \sim U[-\pi, \pi] = U[0, 2\pi],$$

where α is a random variable defining the turnpoint on the search area circumference.

3. **Target Starting Position Is Uniformly Distributed on the Circumference**

a. Creating the Target Motion

Since having shown the equivalence of uniform boundary reflections and selection of uniform circumference nodes, the creation of this target motion is relatively easy. By randomly choosing an $\alpha \sim U[0, 2\pi]$ we may find the (X,Y) coordinate of the target turnpoint on the circumference of the search area with $R_A = 1$ by

$$X = \sin(\alpha)$$
$$Y = \cos(\alpha).$$

Then for each target leg a new value for α is chosen and uniform reflection target motion is generated. A target density may be created by repeating this motion for many targets and recording the target's position after the target is in steady state.

b. Analyzing Target Density

Figure 3.8 shows empirical density and scatter plots for steady state targets starting on the circumference with uniform reflection motion. Notice that this target motion concentrates the target density on the area boundaries. Analysis performed by

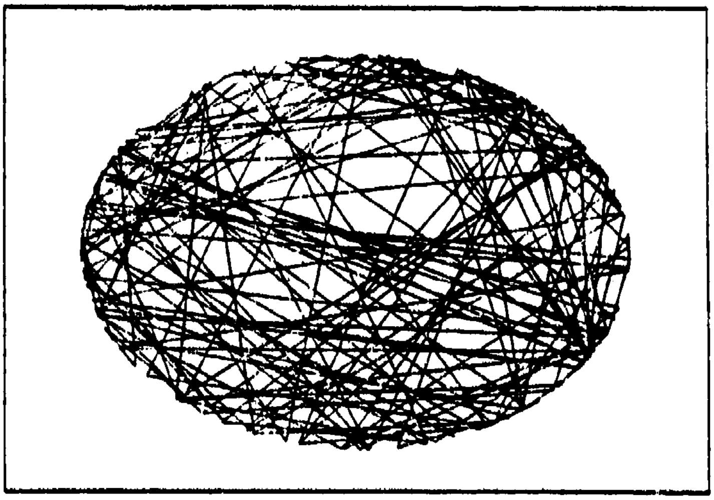

Figure 3.7 Target Paths for Uniform Reflection.

Prof. E. B. Rockower of the Naval Postgraduate School derived the target density as a function of distance from the center of the area for this type of uniform reflection target motion. His calculations are included in Appendix B. The fitted line in Figure 3.8 is a plot of his density function and shows (when appropriately scaled so that both the curves have a density of one on the area boundary) very close agreement with the empirically derived density from this simulation data.

4. Target Starting Position is Uniformly Distributed in the Search Area

a. Creating the Target Motion

The target starts uniformly in the search area and its initial direction of motion is uniformly distributed between 0 and 2π. When the target intersects the boundary its next leg and all subsequent legs are determined by uniform reflections as described above.

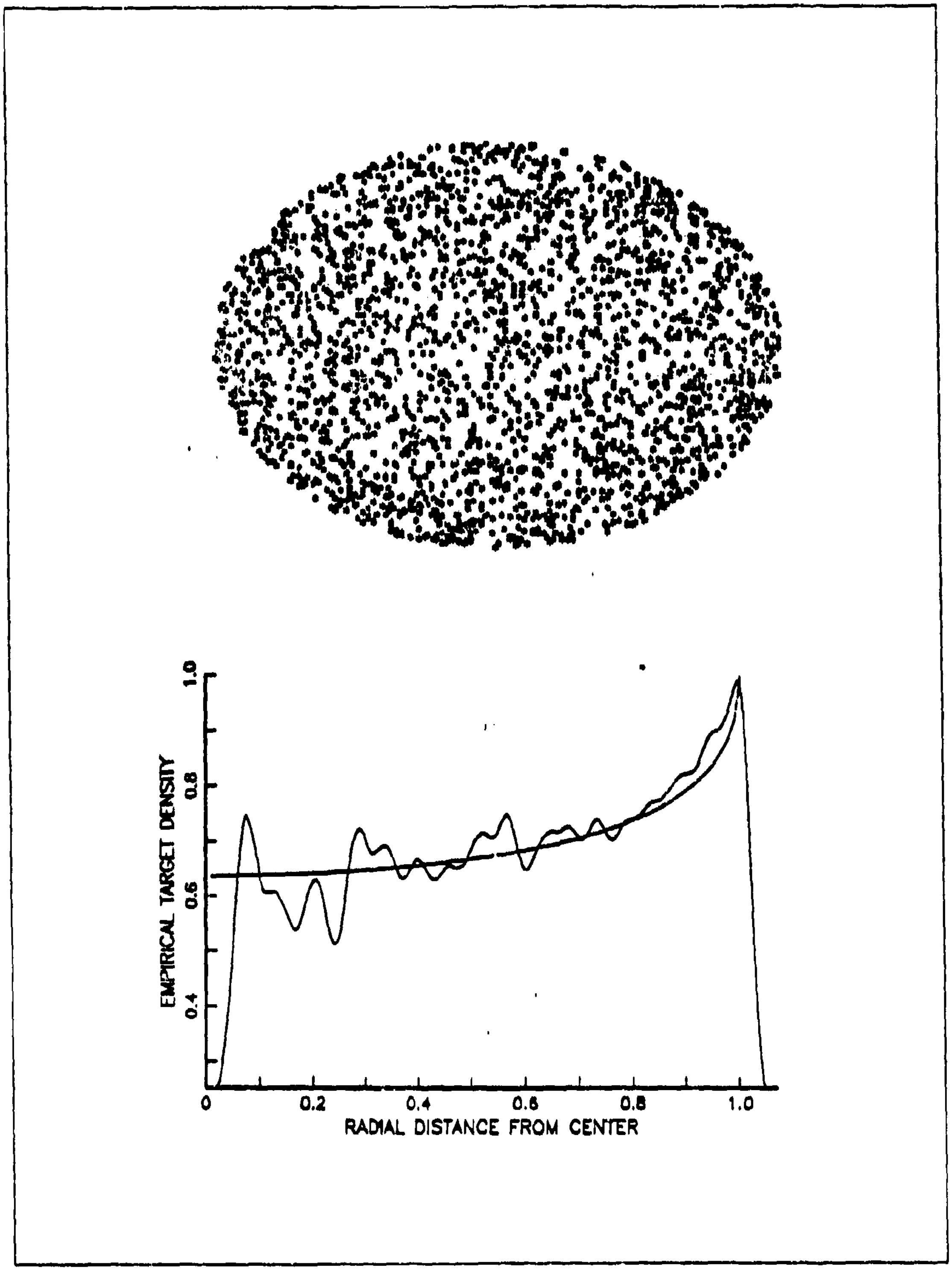

Figure 3.8 Uniform Reflecting Target Starting Uniformly on the Circumference.

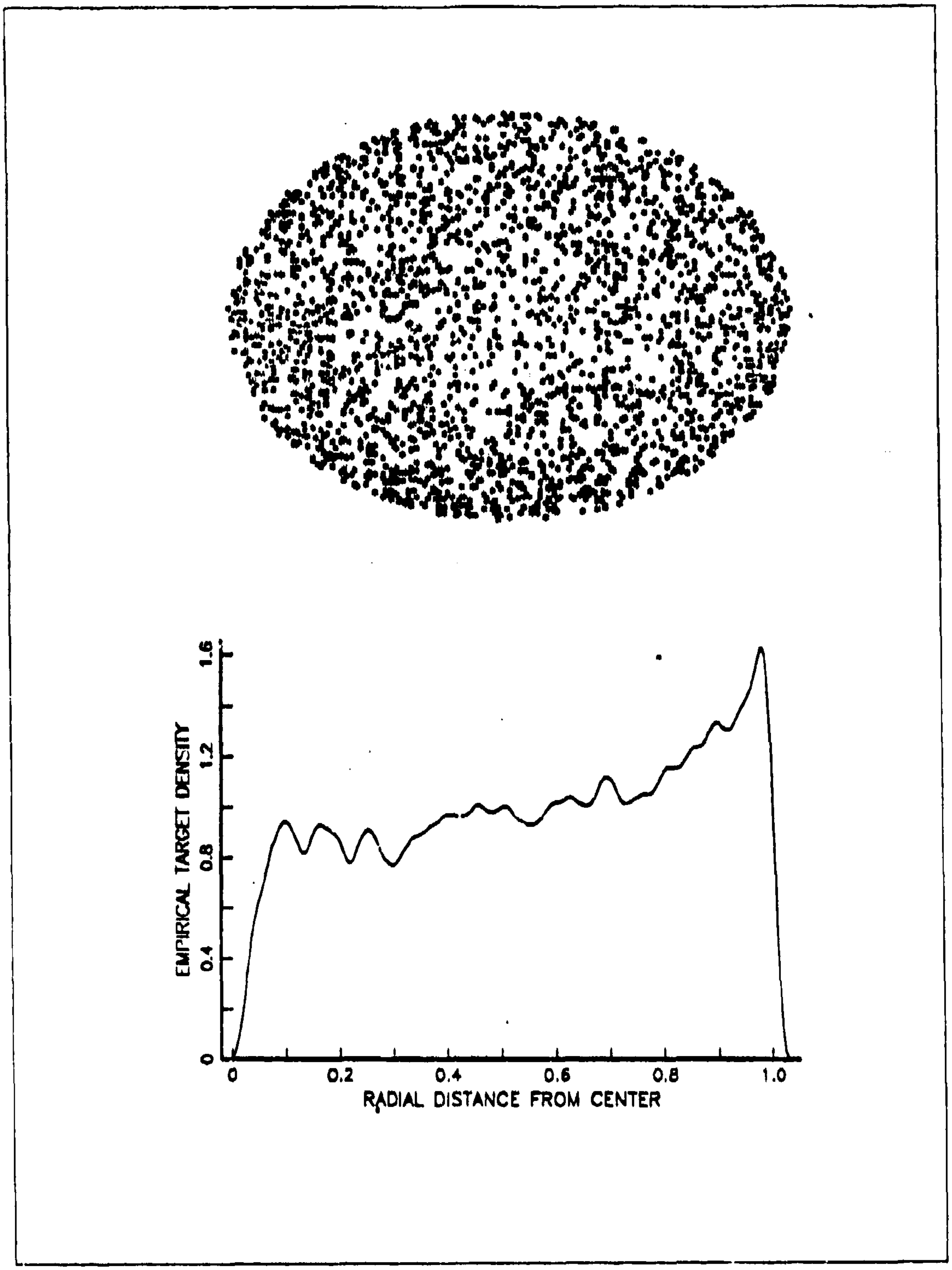

Figure 3.9 Uniform Reflecting Target Starting Uniformly in the Search Area.

b. *Analyzing Target Density*

Figure 3.9 is the empirical density and scatter plots for a steady state uniform reflecting target starting uniformly within the entire search area. This density has the same shape as the density in Figure 3.8 which would be expected since the target loses all memory of where it has been once it encounters a boundary. Therefore, unlike the perfect reflection geometry (Figure 3.5) when the target also starts uniformly about the entire search area, a uniform distribution of targets is *not* obtained. There is at least one other reflection method which should be investigated for completeness.

D. DIFFUSE REFLECTION MODEL

1. Description of Diffuse Reflection

A diffuse reflecting target is similar to a uniform reflecting target. The difference is in the density function of the reflecting angle θ. Whereas $\theta \sim U[-\pi/2, \pi/2]$ for a uniform reflecting target, a diffuse target's angle of reflection has a density function of the form

$$f(\theta) = (1/2)\,\cos(\theta)$$

where the range of θ is also $-\pi/2$ to $\pi/2$. This reflection scheme is illustrated in

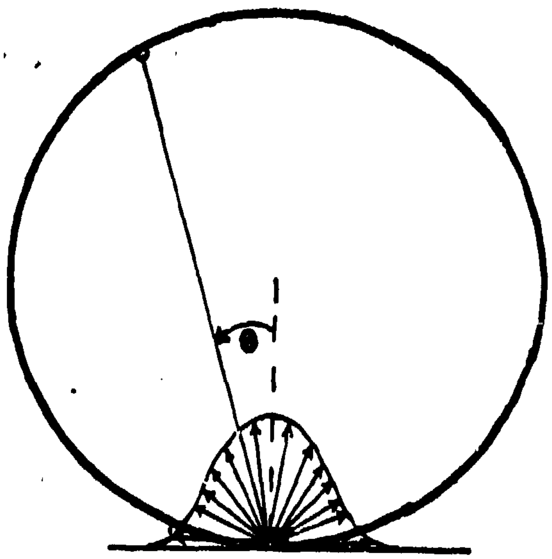

Figure 3.10 Diffuse Reflection Geometry.

Figure 3.10.

The most important characteristic of diffuse reflection is that after a particle undergoes such a reflection, the particle flux density (number of particles per time per length) is constant in every direction. Consider Figure 3.11 where a diffuse reflection occurs anywhere along a small length δ. The probability of a reflected particle crossing line "a" is equal to the probability density on line "a" or approximately

$$\int_{-\delta/2}^{\delta/2} (1/2) \cos(\theta)\, d\theta \approx \delta/2$$

where the limits of integration are the allowable reflection angles expressed in radians. Therefore the flux density through line "a", which is the probability of being reflected through line "a" per length of "a", is $(\delta/2)/\delta = 1/2$. Similarly, the probability of a reflected particle passing through line "b" is approximately

$$\int_{\alpha-\delta/2}^{\alpha+\delta/2} (1/2) \cos(\theta)\, d\theta \approx (1/2)\, \delta \cos(\alpha).$$

But the length of line "b" is $\delta \cos(\alpha)$, so the flux density through line "b" is also 1/2.

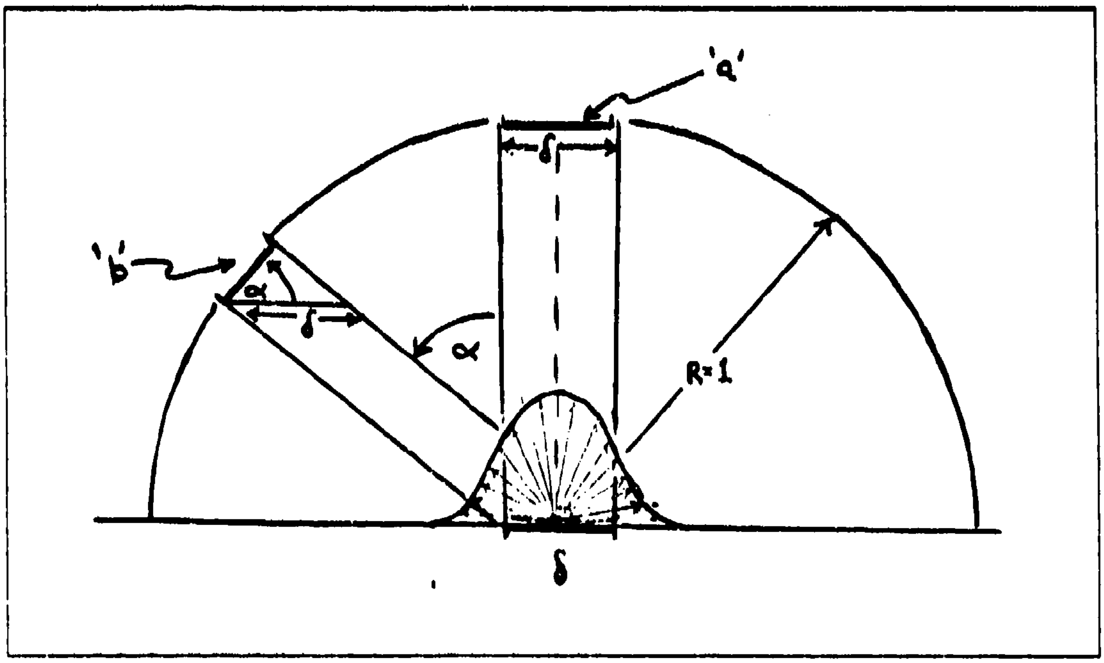

Figure 3.11 Diffuse Particle Reflection.

An alternate way to view these results is to assume that the line "δ" is a diffuse light source. Then the intensity of the light would be constant independent of where the observer stands. A more complete description of diffuse reflection is found in [Ref. 7].

2. **Creating Diffuse Reflection Motion**

The target starts uniformly in the search area and the direction which it initially chooses to move until it encounters a boundary is chosen uniformly. Then the reflection angle of the target, θ, is chosen from the cosine distribution. When the target intersects a boundary again, another value for the random variable θ is selected and the process continues.

3. **Analyzing Target Density**

As shown in Figure 3.12 the density of targets appears uniform. Additional experiments were performed with the target's starting position varying from the center of the area (0.0) to the circumference (1.0). The target densities from these simulations, shown in Figure 3.13, are uniform for each starting position. The conclusion which may be drawn from these results is that *any* deterministic target starting position or distribution of starting positions achieves a uniform steady state distribution.

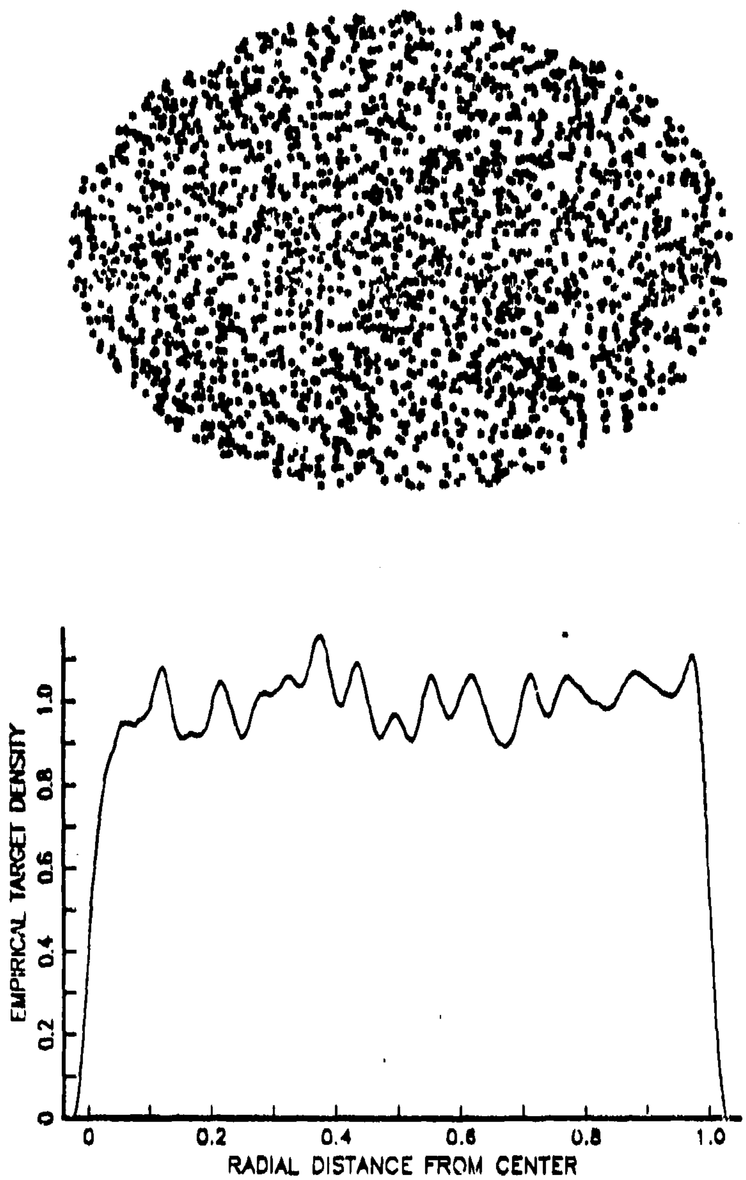

Figure 3.12 Diffuse Reflecting Target
Starting Uniformly in the Entire Search Area.

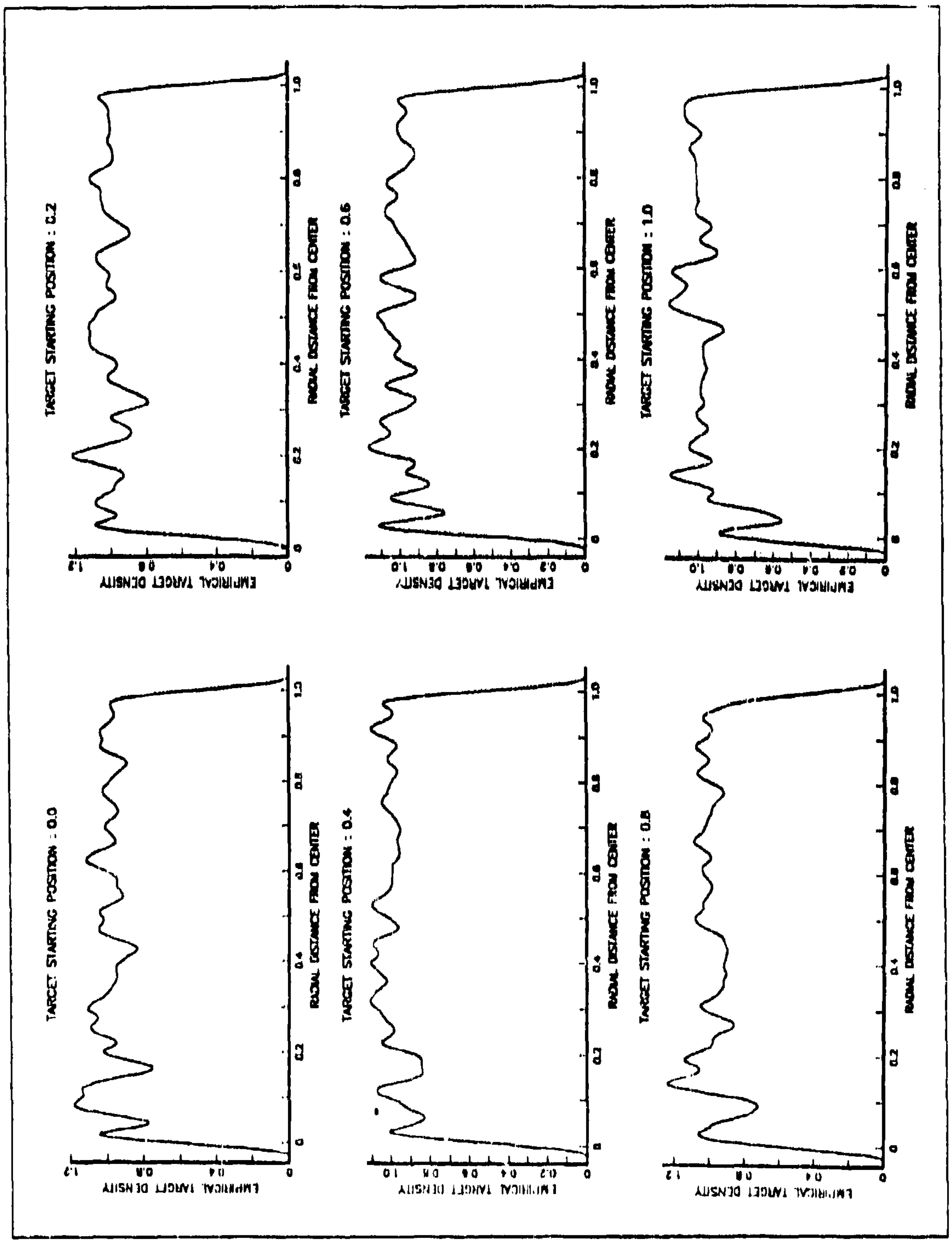

Figure 3.13 Diffuse Reflecting Target Density for Various Target Starting Positions.

IV. APPLICATION OF UNIFORM DISTRIBUTION OF TARGETS TO PND(T)

A. INTRODUCTION

In the last chapter we found that a uniform distribution of targets can be achieved, independent of where the target begins its motion, as long as the target is performing diffuse reflections. We also noted that perfect reflections will provide a uniform distribution of targets if the target's starting position is uniformly distributed in the entire search area. There were no conditions which provided a uniform target density for the uniform reflecting target. In this chapter we will analyze one of the two uniform density target motion models, calculate PND(t) and determine if the model approximates Koopman's random search model.

B. CHOOSING THE MOTION MODEL

As mentioned, we have investigated target motion models with three different reflection patterns, and with the right initial conditions two of these models can create a uniformly distributed target density. One of Koopman's assumptions of random search is that the distribution of targets must be uniform. Following this assumption it would appear that the choice of which reflection pattern to use for analysis does not matter as long as it meets the uniform condition but, as we will see, this is not entirely true. By referring to Figure 3.2 of possible perfect reflection paths, we can see that if the searcher is located in the center of the search area with detection radius R, there will be many target paths for which the searcher will not make a detection. It would not matter how long the simulation ran since the first reflection angle and target leg determines if the searcher will make a detection. Either the target gets detected on the first leg or not at all. Therefore we have shown that a uniform target density, as is the case with the perfect reflecting target, may be a necessary but not a sufficient condition for random search. With the perfect reflecting and uniform target motions eliminated, we will concentrate all further analysis on the diffuse reflecting target.

C. COMPARING THE EXPONENTIAL TO THE PARETO DISTRIBUTION FIT OF PND(T)

Henze found, and this study verified, that the Pareto distribution provided a better fit than the exponential distribution for the time to detection t, when Henze's target motion model was used. The next logical step is to use our diffuse reflecting target and again compare the Pareto and exponential distributions to the computer generated PND(t) using the Chi-Square goodness-of-fit test. As before, we examine the fit for various searcher locations. The results of this experiment are found in Figure 4.1 and suggest the following conclusions:

1) the exponential distribution provides a better fit than the Pareto distribution irrespective of the searcher's location
2) the quality of the exponential fit is also independent of the searcher's position
3) the exponential fit for the diffuse reflecting target is at least an order of magnitude better than the exponential fit for Henze's target motion in Figure 2.1.

This strongly suggests that Henze's results are due to the nonuniform target density of his motion model.

D. PROBABILITY OF NONDETECTION TO TIME T

1. For all Time t

Again we will perform an experiment as in Chapter II by comparing the detection rate for the simulated target with Koopman's predicted detection rate using random search, but this time we use the diffuse reflecting target. Recall that when we used Henze's model, the simulation data did not provide a good estimate of Koopman's random search model.

The diffuse reflecting target motion model experiment included 10,000 iterations at each searcher detection radius R. A plot of the PND(t) vs time (or distance) traveled for values of R from 0.001 to 0.03 is illustrated in Figure 4.2 Notice that the scale of the Y axis is logarithmic, therefore exponential distributions will plot as straight lines. The curves of Figure 4.2 are very nearly linear and the search detection rate is the negative slope of these curves. Table 2 provides a comparison of diffuse reflecting target detection rate determined by least squares fitting of the simulation data with Koopman's random search detection rate. The simulation data suggests the following conclusions:

a) diffuse reflecting target PND(t) gives a reasonable estimate of Koopman's random search formula
b) the estimate improves as R decreases, which implies that the diffuse reflecting target approximates random search in the limit as $R \rightarrow 0$ or when $R << R_A$
c) the observed detection rate does not consistently under or over estimate the detection rate predicted by Koopman which implies that there are no consistent inaccuracies with the model.

2. As Time t $\rightarrow$ 0

For the case of a diffusing target (as opposed to diffuse reflections), James N. Eagle noted that for small t, the decrease in PND(t) is faster than exponential, implying that the curves are not linear as $t \rightarrow 0$ [Ref. 2: page 47]. Figure 4.3 is an enlargement of the upper left corner of Figure 4.2. Notice that the phenomenon which Eagle observed is not present in this target motion model. No explanation of this

TABLE 2

COMPARISON OF DETECTION RATES FOR RANDOM SEARCH AND HENZE'S REVISED MODEL WITH DIFFUSE REFLECTIONS

Detection Radius R	(RS) Random Search Detection Rates	(S) Simulation Detection Rates	$\frac{S}{RS}$
0.003	1.910E-3	1.863E-3	0.98
0.006	3.820E-3	3.782E-3	0.99
0.01	0.006366	0.006435	1.01
0.03	0.01910	0.01935	1.01
0.05	0.03183	0.03310	1.04
0.10	0.06366	0.06854	1.08
0.15	0.09549	0.10729	1.12
0.20	0.12732	0.15152	1.19

effect is offered here except to remark that the absence of this effect permits the diffuse reflecting target model to more closely approximate the exponential distribution and provide a better tool for analysis.

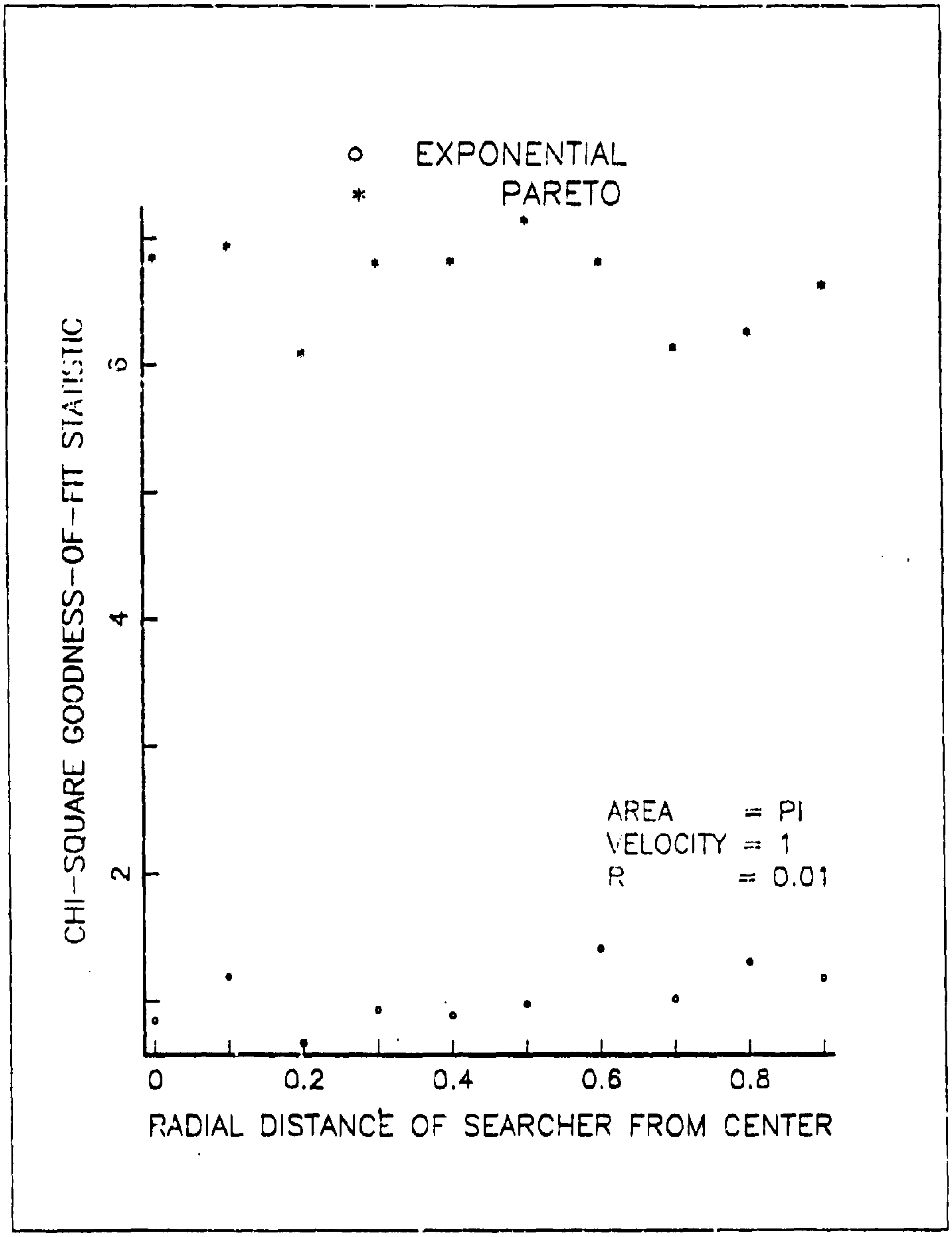

Figure 4.1 Comparison of Pareto vs Exponential Fit for different Searcher Locations.

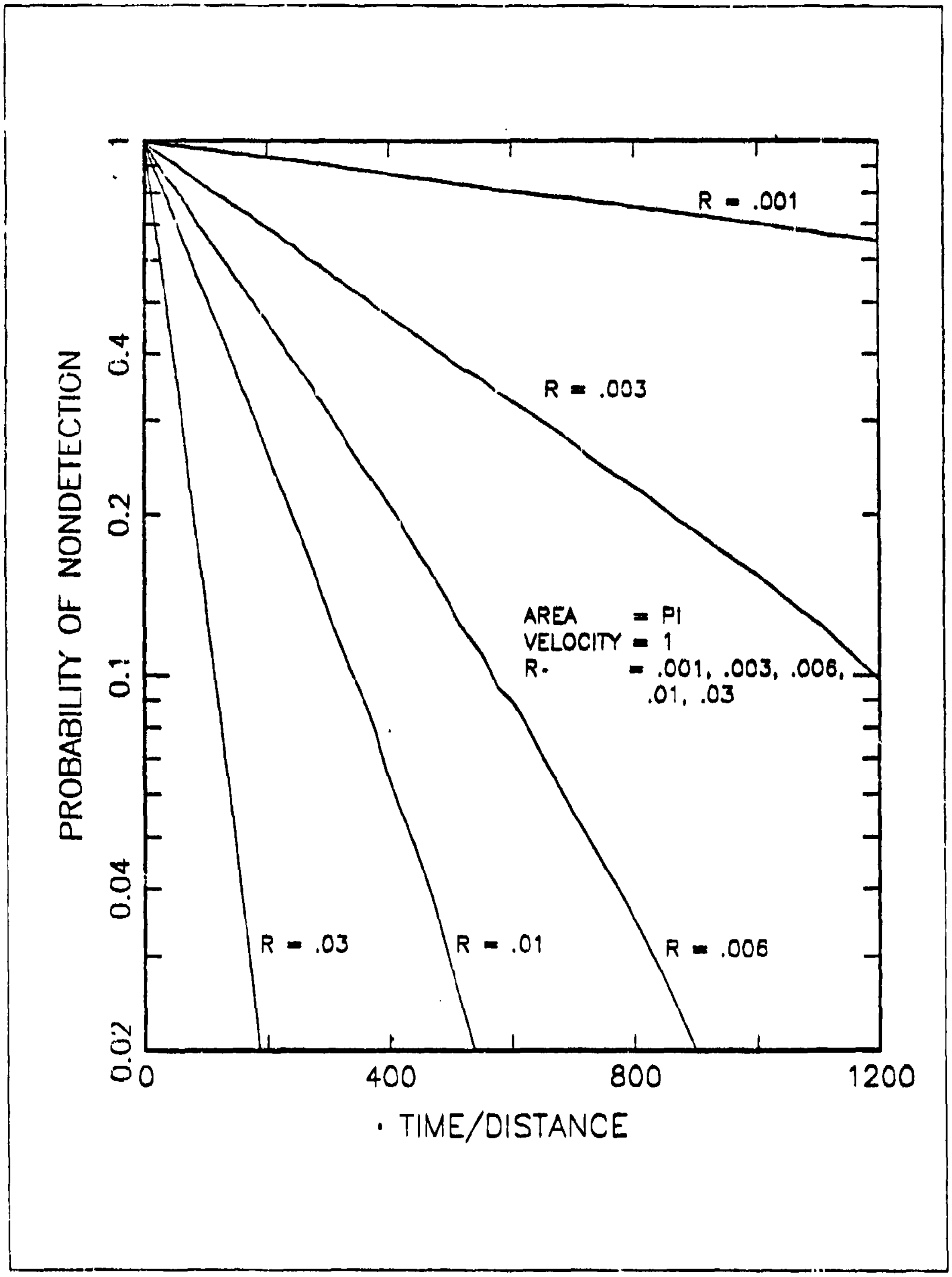

Figure 4.2 Probability of Non Detection by Time t for Uniformly Distributed Targets.

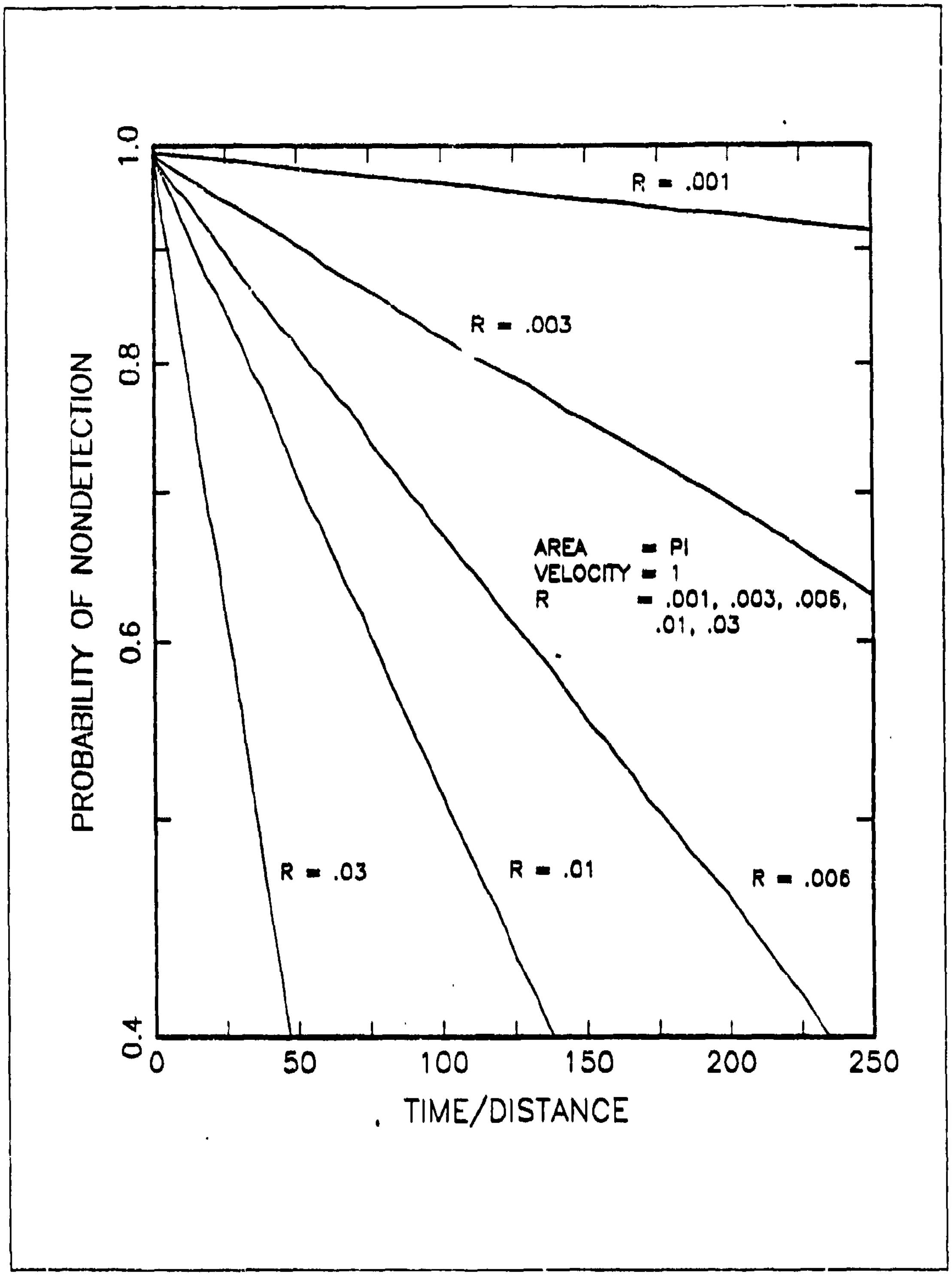

Figure 4.3 Probability of Non Detection by Time t for Uniformly Distributed Targets.

V. CONCLUSIONS

This thesis was motivated by a study performed by COMSUBPAC which concluded that PND(t) did not have an exponential form, contradicting expectations and some theory. In the process of investigating this problem, we discovered that the target density for the COMSUBPAC model was not uniform, which is an assumption made by Koopman in his development of the random search formula. Proceeding on the assumption that the nonuniform target density was the cause of the nonexponential distribution, we began investigating ways to reclaim a uniform distribution of targets in the COMSUBPAC model. The basic characteristics of the Henze model were retained due to its desirable simplicity.

A diffuse reflecting target was found to provide the best reflecting scheme to restore the uniform target density to the Henze model. An exponential PND(t) was achieved with this new motion model which, for detectors with a relatively small size, very closely approximated the detection rates predicted by Koopman's random search formula.

APPENDIX A
FORTRAN PROGRAM FOR REFLECTING TARGETS

1. DESCRIPTION OF THE VARIABLES

Alpha : Uniformly distributed random variable between 0 and 2π

Anormal : Angle of the surface normal at the given point

Beta : Uniformly distributed random variable between 0 and 2π

Detect : Array of times to detection

Detrng : Definite detection range of the searcher

Dist : Distance between two points

Dscrmt : Discriminate from the solution to the simultaneous equations for searcher's detection disk and target path

Fposit : Final position of target's leg

Iposit : Initial position of target's leg

Nreps : Number of repetitions for the search encounter

Secant : Distance between Iposit and Fposit or the length of the target's path

Target : Coordinates of the searcher

Totdis : Total distance traveled by the target

Unifrm : Uniform random number between 0 and 1

Velcty : Velocity of the target

2. FORTRAN PROGRAM 'NODE'

```
      REAL FPOSIT(2),UNIFRM(9),IPOSIT(2),DETECT(9000),START(2),TARGET(2)
*********************************************************************
* INPUT INITIAL CONDITIONS
*********************************************************************
* ISTART .EQ. 1 FOR TARGET STARTING UNIFORMLY IN THE ENTIRE AREA.
* ISTART .NE. 1 FOR TARGET STARTING UNIFORMLY ON THE CIRCUMFERENCE
* OF THE SEARCH AREA
*
      ISTART=0
*
* IREFLT = 0 FOR DIFFUSE TARGET REFLECTION
* IREFLT = 1 FOR UNIFORM TARGET REFLECTION
* IREFLT = 2 FOR PERFECT TARGET REFLECTION
* IREFLT = 3 FOR HENZE TARGET MOTION
*
      IREFLT=0
*
* NREPS: NUMBER OF REPETITIONS OF THE SEARCH
*
      NREPS=10000
*
```

```
* DETRNG: DEFINITE DETECTION RANGE OF THE SEARCHER 0<DETRNG<1
*
      DETRNG=0.001
*
* VELCTY: VELOCITY OF THE TARGET
*
      VELCTY=1
*
      PI=3.1415927
      IX=10099
      TARGET(1)=0.0
      TARGET(2)=0.0
*********************************************************************
* BEGIN REPETITION OF A SEARCH
*********************************************************************
      DO 300 I=1,NREPS
      TOTDIS=0
*********************************************************************
* ESTABLISH INITIAL TARGET STARTING POSITIONS
*********************************************************************
          IF (ISTART .EQ. 1 ) THEN
******  ENTIRE AREA START
                  CALL SEARCH (START,IX)
                  CALL DSTNCE(START,TARGET,DIST)
                  IF (DIST .LE. DETRNG) GO TO 200
                  CALL LRND(IX,UNIFRM,1,1,0)
                  ALPHA=2*PI*UNIFRM(1)
                  IPOSIT(1)=START(1)+2*COS(ALPHA)
                  IPOSIT(2)=START(2)+2*SIN(ALPHA)
                  FPOSIT(1)=START(1)-2*COS(ALPHA)
                  FPOSIT(2)=START(2)-2*SIN(ALPHA)
                  CALL DISCRM  (IPOSIT,FPOSIT,TARGET,1.0,DSCRMT,A,B)
                  CALL INTSCT(DSCRMT,A,B,S,T)
                  FPOSIT(1)=IPOSIT(1)+S*(FPOSIT(1)-IPOSIT(1))
                  FPOSIT(2)=IPOSIT(2)+S*(FPOSIT(2)-IPOSIT(2))
                  IPOSIT(1)=START(1)
                  IPOSIT(2)=START(2)
******  HENZE MOTION MODEL
                  IF (IREFLT .EQ. 3) THEN CALL SEARCH (FPOSIT,IX)
          ELSE
******  UNIFORM CIRCUMFERENCE START
                  CALL LRND(IX,UNIFRM,1,1,0)
                  ALPHA=2*PI*UNIFRM(1)
                  IPOSIT(1)=COS(ALPHA)
                  IPOSIT(2)=SIN(ALPHA)
                  IF (IREFLT .EQ. 0) THEN
****** DIFFUSE REFLECTING TARGET
                         CALL DIFUSE(IPOSIT,FPOSIT,IX,PI,TARGET)
                  ELSE
****** PERFECT OR UNIFORM REFLECTING TARGET
                         CALL LRND(IX,UNIFRM,1,1,0)
                         ALPHA=2*PI*UNIFRM(1)
                         FPOSIT(1)=COS(ALPHA)
                         FPOSIT(2)=SIN(ALPHA)
```

```
                END IF
          END IF
****** TARGET SEARCH LOOP
100    CALL DSTNCE (IPOSIT,FPOSIT,SECANT)
       CALL DISCRM  (IPOSIT,FPOSIT,TARGET,DETRNG,DSCRMT,A,B)
       IF (DSCRMT .LE. 0) THEN
***** TARGET PATH DOES NOT INTERSECT SEARCHER'S DISK
          TOTDIS=TOTDIS+SECANT*VELCTY
          IF (IREFLT .EQ. 0) THEN
****** DIFFUSE REFLECTING TARGET
                IPOSIT(1)=FPOSIT(1)
                IPOSIT(2)=FPOSIT(2)
                CALL DIFUSE(IPOSIT,FPOSIT,IX,PI,TARGET)
          ELSE IF (IREFLT .EQ. 1) THEN
****** UNIFORM REFLECTING TARGET
               CALL LRND(IX,UNIFRM,1,1,0)
               ALPHA=2*PI*UNIFRM(1)
               IPOSIT(1)=FPOSIT(1)
               IPOSIT(2)=FPOSIT(2)
               FPOSIT(1)=COS(ALPHA)
               FPOSIT(2)=SIN(ALPHA)
          ELSE IF (IREFLT .EQ. 2) THEN
****** PERFECT REFLECTING TARGET
                CALL REFLCT (IPOSIT,FPOSIT)
******  HENZE MOTION MODEL
          ELSE IF (IREFLT .EQ. 3) THEN
               IPOSIT(1)=FPOSIT(1)
               IPOSIT(2)=FPOSIT(2)
               CALL SEARCH (FPOSIT,IX)
          END IF
       ELSE
**********************************************************************
* THE TARGET PATH DOES INTERSECT THE SEARCHER'S DETECTION CIRCLE
* BUT NOT NECESSARILY BETWEEN IPOSIT AND FPOSIT
**********************************************************************
         CALL INTSCT(DSCRMT,A,B,S,T)
         IF ((T .GE. 0) .AND. (T .LE. 1)) THEN
**********************************************************************
* THE TARGET PATH INTERSECTS THE SEARCHER'S DETECTION CIRCLE
* BETWEEN IPOSIT AND FPOSIT
**********************************************************************
                       TOTDIS=TOTDIS+SECANT*T*VELCTY
       GO TO 200
           ELSE
**********************************************************************
* THE TARGET PATH INTERSECTS THE SEARCHER'S DETECTION CIRCLE
* BUT NOT BETWEEN IPOSIT AND FPOSIT
* UPDATE NEW POSITION AND TOTAL DISTANCE TRAVELED
**********************************************************************
                TOTDIS=TOTDIS+SECANT*VELCTY
                IF (IREFLT .EQ. 0) THEN
****** DIFFUSE REFLECTING TARGET
```

```
                    IPOSIT(1)=FPOSIT(1)
                    IPOSIT(2)=FPOSIT(2)
                    CALL DIFUSE(IPOSIT,FPOSIT,IX,PI,TARGET)
              ELSE IF (IREFLT .EQ. 1) THEN
****** UNIFORM REFLECTING TARGET
                    CALL LRND(IX,UNIFRM,1,1,0)
                    ALPHA=2*PI*UNIFRM(1)
                    IPOSIT(1)=FPOSIT(1)
                    IPOSIT(2)=FPOSIT(2)
                    FPOSIT(1)=COS(ALPHA)
                    FPOSIT(2)=SIN(ALPHA)
              ELSE IF (IREFLT .EQ. 2) THEN
****** PERFECT REFLECTING TARGET
              CALL REFLCT (IPOSIT,FPOSIT)
******  HENZE MOTION MODEL
              ELSE IF (IREFLT .EQ. 3) THEN
                   IPOSIT(1)=FPOSIT(1)
                   IPOSIT(2)=FPOSIT(2)
                   CALL SEARCH (FPOSIT,IX)
              END IF
           END IF
        END IF
        GO TO 100
200     DETECT(I)=TOTDIS
300     CONTINUE
        WRITE(69,450)(DETECT(L),L=1,NREPS)
450     FORMAT(10(F8.2))
500     STOP
600     END
**********************************************************************
* FINDS THE DISTANCE BETWEEN TWO POINTS
**********************************************************************
        SUBROUTINE DSTNCE(POINTA,POINTB,DIST)
        REAL POINTA(2),POINTB(2)
        DIST=SQRT((POINTA(1)-POINTB(1))**2+(POINTA(2)-POINTB(2))**2)
10      RETURN
20      END

**********************************************************************
* FINDS RANDOM CO-ORDINATE WITHIN THE SEARCH AREA
**********************************************************************
        SUBROUTINE SEARCH(POSIT,IX)
        REAL POSIT(2),PI,TARGET(2),UNIFRM(2)
        PI=3.1415927
        TARGET(1)=0
        TARGET(2)=0
10      CALL SRND(IX,UNIFRM,2,1,0)
        POSIT(1)=UNIFRM(1)*2-1
        POSIT(2)=UNIFRM(2)*2-1
        CALL DSTNCE(POSIT,TARGET,RADIUS)
        IF (RADIUS .GT. 1) GO TO 10
20      RETURN
30      END
```

```
*******************************************************************
* FINDS DISCRIMANT OF TARGET'S PATH WITH SEARCHER'S DETECTION CIRCLE
*******************************************************************
       SUBROUTINE DISCRM(IPOSIT,FPOSIT,TARGET,DETRNG,DSCRMT,A,B)
       REAL  IPOSIT(2),FPOSIT(2),TARGET(2)
       A=(IPOSIT(1)-FPOSIT(1))**2+(IPOSIT(2)-FPOSIT(2))**2
       B=2*((FPOSIT(1)-IPOSIT(1))*(IPOSIT(1)-TARGET(1))+
      1    (FPOSIT(2)-IPOSIT(2))*(IPOSIT(2)-TARGET(2)))
       C=(IPOSIT(1)-TARGET(1))**2+
      1  (IPOSIT(2)-TARGET(2))**2-DETRNG**2
       DSCRMT=B**2-4*A*C
20     RETURN
30     END
*******************************************************************
* DETERMINES POINT OF INTERSECTION OF
*TARGET'S PATH WITH SEARCHER'S DETECTION CIRCLE
*******************************************************************
       SUBROUTINE INTSCT(DSCRMT,A,B,S,T)
       S=(-B+SQRT(DSCRMT))/2/A
       T=(-B-SQRT(DSCRMT))/2/A
10     RETURN
20     END
*******************************************************************
* DETERMINES NEXT SEARCH LEG END POINT FOR DIFFUSE REFLECTION
*******************************************************************
       SUBROUTINE DIFUSE(IPOSIT,FPOSIT,IX,PI,TARGET)
       REAL IPOSIT(2),FPOSIT(2),UNIFRM(2),TARGET(2),PI
       CALL LRND(IX,UNIFRM,1,1,0)
       ANORML=ATAN(IPOSIT(2)/IPOSIT(1))
       IF (IPOSIT(1) .GE. 0) ANORML=PI+ANORML
       BETA=ANORML+ASIN(2*UNIFRM(1)-1)
       FPOSIT(1)=IPOSIT(1)+2*COS(BETA )
       FPOSIT(2)=IPOSIT(2)+2*SIN(BETA )
       CALL DISCRM(IPOSIT,FPOSIT,TARGET,1.0,DSCRMT,A,B)
       CALL INTSCT(DSCRMT,A,B,S,T)
       FPOSIT(1)=IPOSIT(1)+S*(FPOSIT(1)-IPOSIT(1))
       FPOSIT(2)=IPOSIT(2)+S*(FPOSIT(2)-IPOSIT(2))
10     RETURN
20     END
*******************************************************************
* FINDS THE NEXT LEG END POINT ASSUMING PERFECT REFLECTION
*******************************************************************
       SUBROUTINE REFLCT(IPOSIT,FPOSIT)
       REAL IPOSIT(2),FPOSIT(2)
       CROSS=-FPOSIT(1)*(FPOSIT(2)-IPOSIT(2))+FPOSIT(2)*(FPOSIT(1)
      1-IPOSIT(1))
       DOT=-((IPOSIT(1)-FPOSIT(1))*FPOSIT(1)+
      1(IPOSIT(2)-FPOSIT(2))*FPOSIT(2))
       PERP1=SQRT((IPOSIT(1)-FPOSIT(1))**2+(IPOSIT(2)-FPOSIT(2))**2)
       PERP2=SQRT(FPOSIT(1)**2+FPOSIT(2)**2)
       THETA=ACOS(DOT/(PERP1*PERP2))
       IF (CROSS .GT. 0) THEN
          V1=FPOSIT(2)*SIN(THETA)-FPOSIT(1)*COS(THETA)
```

```
        V2=-(FPOSIT(1)*SIN(THETA)+FPOSIT(2)*COS(THETA))
      ELSE
        V1=-(FPOSIT(1)*COS(THETA)+FPOSIT(2)*SIN(THETA))
        V2=FPOSIT(1)*SIN(THETA)-FPOSIT(2)*COS(THETA)
      END IF
      Q1=V1**2+V2**2
      Q2=-2*(FPOSIT(1)*V1+FPOSIT(2)*V2)
      IPOSIT(1)=FPOSIT(1)
      IPOSIT(2)=FPOSIT(2)
      FPOSIT(1)=FPOSIT(1)+Q2*V1/Q1
      FPOSIT(2)=FPOSIT(2)+Q2*V2/Q1
20    RETURN
30    END
```

APPENDIX B
ANALYTIC MODEL OF UNIFORM REFLECTING TARGET DENSITY

E. B. Rockower of the Naval Postgraduate School, Monterey, California, derived the target density for a uniform reflecting target. His initial assumptions were that

1) the target starts uniformly on the circumference of a circular area and
2) the target density is equal to one on the area boundary.

Refer to Figure B.1, where R_A is the radius of the search area and the uniform

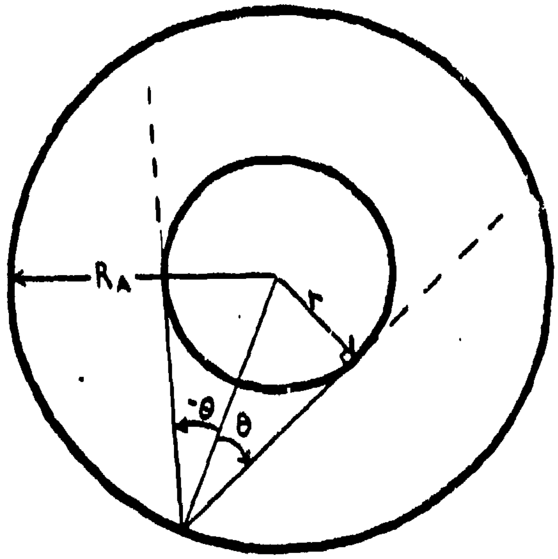

Figure B.1 Uniform Reflection Geometry.

reflection angle is $\theta \sim U[-\pi/2, \pi/2]$. He calculates as a function of r the target density, $\rho(r)$, by assuming that the line density of targets on the smaller circle between $-\theta$ and θ is proportional to area density of targets. Then the line density of the smaller circle between $-\theta$ and θ is

$$\int_{-\arcsin(r/R_A)}^{\arcsin(r/R_A)} (1/\pi)\, d\theta = (2/\pi)\, \mathrm{ARCSIN}\,(r/R_A)\,.$$

Therefore it follows that the area density would be

$$\rho(r) = (2R_A/(\pi r))\, \mathrm{ARCSIN}\,(r/R_A)\,.$$

To find the target density at the center we take the limit of $\rho(r)$ as $r \to 0$

$$\rho(r=0) = 2\pi\,.$$

LIST OF REFERENCES

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